

ANSWERS

EXERCISE 7.1

1. $-\frac{1}{2}\cos 2x$

2. $\frac{1}{3}\sin 3x$

3. $\frac{1}{2}e^{2x}$

4. $\frac{1}{3a}(ax+b)^3$

5. $-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}$

6. $\frac{4}{3}e^{3x} + x + C$

7. $\frac{x^3}{3} - x + C$

8. $\frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

9. $\frac{2}{3}x^3 + e^x + C$

10. $\frac{x^2}{2} + \log|x| - 2x + C$

11. $\frac{x^2}{2} + 5x + \frac{4}{x} + C$

12. $\frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$

13. $\frac{x^3}{3} + x + C$

14. $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$

15. $\frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$

16. $x^2 - 3\sin x + e^x + C$

17. $\frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{\frac{3}{2}} + C$

18. $\tan x + \sec x + C$

19. $\tan x - x + C$

20. $2 \tan x - 3 \sec x + C$

21. C

22. A

EXERCISE 7.2

1. $\log(1+x^2) + C$

2. $\frac{1}{3}(\log|x|)^3 + C$

3. $\log|1+\log x| + C$

4. $\cos(\cos x) + C$

5. $-\frac{1}{4a}\cos 2(ax+b) + C$

6. $\frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

7. $\frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

8. $\frac{1}{6}(1+2x^2)^{\frac{3}{2}} + C$ 9. $\frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C$ 10. $2\log|\sqrt{x}-1|+C$

11. $\frac{2}{3}\sqrt{x+4}(x-8)+C$

12. $\frac{1}{7}(x^3-1)^{\frac{7}{3}} + \frac{1}{4}(x^3-1)^{\frac{4}{3}} + C$ 13. $-\frac{1}{18(2+3x^3)^2}+C$

14. $\frac{(\log x)^{1-m}}{1-m}+C$ 15. $-\frac{1}{8}\log|9-4x^2|+C$ 16. $\frac{1}{2}e^{2x+3}+C$

17. $-\frac{1}{2e^{x^2}}+C$ 18. $e^{\tan^{-1}x}+C$ 19. $\log(e^x+e^{-x})+C$

20. $\frac{1}{2}\log(e^{2x}+e^{-2x})+C$ 21. $\frac{1}{2}\tan(2x-3)-x+C$

22. $-\frac{1}{4}\tan(7-4x)+C$ 23. $\frac{1}{2}(\sin^{-1}x)^2+C$

24. $\frac{1}{2}\log|2\sin x+3\cos x|+C$ 25. $\frac{1}{(1-\tan x)}+C$

26. $2\sin\sqrt{x}+C$ 27. $\frac{1}{3}(\sin 2x)^{\frac{3}{2}}+C$ 28. $2\sqrt{1+\sin x}+C$

29. $\frac{1}{2}(\log \sin x)^2+C$ 30. $-\log|1+\cos x|+C$ 31. $\frac{1}{1+\cos x}+C$

32. $\frac{x}{2}-\frac{1}{2}\log|\cos x+\sin x|+C$ 33. $\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$

34. $2\sqrt{\tan x}+C$ 35. $\frac{1}{3}(1+\log x)^3+C$ 36. $\frac{1}{3}(x+\log x)^3+C$

37. $-\frac{1}{4}\cos(\tan^{-1}x^4)+C$ 38. D

39. B

EXERCISE 7.3

1. $\frac{x}{2} - \frac{1}{8} \sin(4x+10) + C$

2. $-\frac{1}{14} \cos 7x + \frac{1}{2} \cos x + C$

3. $\frac{1}{4} \left[\frac{1}{12} \sin 12x + x + \frac{1}{8} \sin 8x + \frac{1}{4} \sin 4x \right] + C$

4. $-\frac{1}{2} \cos(2x+1) + \frac{1}{6} \cos^3(2x+1) + C$

5. $\frac{1}{6} \cos^6 x - \frac{1}{4} \cos^4 x + C$

6. $\frac{1}{4} \left[\frac{1}{6} \cos 6x - \frac{1}{4} \cos 4x - \frac{1}{2} \cos 2x \right] + C$

7. $\frac{1}{2} \left[\frac{1}{4} \sin 4x - \frac{1}{12} \sin 12x \right] + C$

8. $2 \tan \frac{x}{2} - x + C$

9. $x - \tan \frac{x}{2} + C$

10. $\frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$

11. $\frac{3x}{8} + \frac{1}{8} \sin 4x + \frac{1}{64} \sin 8x + C$

12. $x - \sin x + C$

13. $2(\sin x + x \cos x) + C$

14. $-\frac{1}{\cos x + \sin x} + C$

15. $\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + C$

16. $\frac{1}{3} \tan^3 x - \tan x + x + C$

17. $\sec x - \operatorname{cosec} x + C$

18. $\tan x + C$

19. $\log |\tan x| + \frac{1}{2} \tan^2 x + C$

20. $\log |\cos x + \sin x| + C$

21. $\frac{\pi x}{2} - \frac{x^2}{2} + C$

22. $\frac{1}{\sin(a-b)} \log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| + C$

23. A

24. B

EXERCISE 7.4

1. $\tan^{-1} x^3 + C$

2. $\frac{1}{2} \log \left| 2x + \sqrt{1+4x^2} \right| + C$

3. $\log \left| \frac{1}{2-x+\sqrt{x^2-4x+5}} \right| + C$ **4.** $\frac{1}{5} \sin^{-1} \frac{5x}{3} + C$

5. $\frac{3}{2\sqrt{2}} \tan^{-1} \sqrt{2} x^2 + C$ **6.** $\frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C$

7. $\sqrt{x^2-1} - \log \left| x + \sqrt{x^2-1} \right| + C$ **8.** $\frac{1}{3} \log \left| x^3 + \sqrt{x^6+a^6} \right| + C$

9. $\log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C$ **10.** $\log \left| x+1+\sqrt{x^2+2x+2} \right| + C$

11. $\frac{1}{6} \tan^{-1} \left(\frac{3x+1}{2} \right) + C$ **12.** $\sin^{-1} \left(\frac{x+3}{4} \right) + C$

13. $\log \left| x - \frac{3}{2} + \sqrt{x^2-3x+2} \right| + C$ **14.** $\sin^{-1} \left(\frac{2x-3}{\sqrt{41}} \right) + C$

15. $\log \left| x - \frac{a+b}{2} + \sqrt{(x-a)(x-b)} \right| + C$

16. $2\sqrt{2x^2+x-3} + C$ **17.** $\sqrt{x^2-1} + 2 \log \left| x + \sqrt{x^2-1} \right| + C$

18. $\frac{5}{6} \log |3x^2+2x+1| - \frac{11}{3\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) + C$

19. $6\sqrt{x^2-9x+20} + 34 \log \left| x - \frac{9}{2} + \sqrt{x^2-9x+20} \right| + C$

20. $-\sqrt{4x-x^2} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) + C$

21. $\sqrt{x^2+2x+3} + \log \left| x+1+\sqrt{x^2+2x+3} \right| + C$

22. $\frac{1}{2} \log |x^2-2x-5| + \frac{2}{\sqrt{6}} \log \left| \frac{x-1-\sqrt{6}}{x-1+\sqrt{6}} \right| + C$

23. $5\sqrt{x^2 + 4x + 10} - 7 \log|x + 2 + \sqrt{x^2 + 4x + 10}| + C$

24. B

25. B

EXERCISE 7.5

1. $\log \frac{(x+2)^2}{|x+1|} + C$

2. $\frac{1}{6} \log \left| \frac{x-3}{x+3} \right| + C$

3. $\log|x-1| - 5 \log|x-2| + 4 \log|x-3| + C$

4. $\frac{1}{2} \log|x-1| - 2 \log|x-2| + \frac{3}{2} \log|x-3| + C$

5. $4 \log|x+2| - 2 \log|x+1| + C$ 6. $\frac{x}{2} + \log|x| - \frac{3}{4} \log|1-2x| + C$

7. $\frac{1}{2} \log|x-1| - \frac{1}{4} \log(x^2 + 1) + \frac{1}{2} \tan^{-1} x + C$

8. $\frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + C$ 9. $\frac{1}{2} \log \left| \frac{x+1}{x-1} \right| - \frac{4}{x-1} + C$

10. $\frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{12}{5} \log|2x+3| + C$

11. $\frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C$

12. $\frac{x^2}{2} + \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$

13. $-\log|x-1| + \frac{1}{2} \log(1+x^2) + \tan^{-1} x + C$

14. $3 \log|x+2| + \frac{7}{x+2} + C$ 15. $\frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C$

16. $\frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C$

17. $\log \left| \frac{2-\sin x}{1-\sin x} \right| + C$

18. $x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3 \tan^{-1} \frac{x}{2} + C$ 19. $\frac{1}{2} \log \left(\frac{x^2 + 1}{x^2 + 3} \right) + C$

20. $\frac{1}{4} \log \left| \frac{x^4 - 1}{x^4} \right| + C$

22. B

21. $\log \left(\frac{e^x - 1}{e^x} \right) + C$

23. A

EXERCISE 7.6

1. $-x \cos x + \sin x + C$

2. $-\frac{x}{3} \cos 3x + \frac{1}{9} \sin 3x + C$

3. $e^x (x^2 - 2x + 2) + C$

4. $\frac{x^2}{2} \log x - \frac{x^2}{4} + C$

5. $\frac{x^2}{2} \log 2x - \frac{x^2}{4} + C$

6. $\frac{x^3}{3} \log x - \frac{x^3}{9} + C$

7. $\frac{1}{4} (2x^2 - 1) \sin^{-1} x + \frac{x\sqrt{1-x^2}}{4} + C$

8. $\frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C$

9. $(2x^2 - 1) \frac{\cos^{-1} x}{4} - \frac{x}{4} \sqrt{1-x^2} + C$

10. $(\sin^{-1} x)^2 x + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C$

11. $-\sqrt{1-x^2} \cos^{-1} x + x + C$

12. $x \tan x + \log |\cos x| + C$

13. $x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + C$

14. $\frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C$

15. $\left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$

16. $e^x \sin x + C$

17. $\frac{e^x}{1+x} + C$

18. $e^x \tan \frac{x}{2} + C$

19. $\frac{e^x}{x} + C$

20. $\frac{e^x}{(x-1)^2} + C$

21. $\frac{e^{2x}}{5} (2 \sin x - \cos x) + C$

22. $2x \tan^{-1} x - \log(1+x^2) + C$

23. A

24. B

EXERCISE 7.7

1. $\frac{1}{2}x\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$
2. $\frac{1}{4}\sin^{-1}2x + \frac{1}{2}x\sqrt{1-4x^2} + C$
3. $\frac{(x+2)}{2}\sqrt{x^2+4x+6} + \log|x+2+\sqrt{x^2+4x+6}| + C$
4. $\frac{(x+2)}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log|x+2+\sqrt{x^2+4x+1}| + C$
5. $\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right) + \frac{x+2}{2}\sqrt{1-4x-x^2} + C$
6. $\frac{(x+2)}{2}\sqrt{x^2+4x-5} - \frac{9}{2}\log|x+2+\sqrt{x^2+4x-5}| + C$
7. $\frac{(2x-3)}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right) + C$
8. $\frac{2x+3}{4}\sqrt{x^2+3x} - \frac{9}{8}\log|x+\frac{3}{2}+\sqrt{x^2+3x}| + C$
9. $\frac{x}{6}\sqrt{x^2+9} + \frac{3}{2}\log|x+\sqrt{x^2+9}| + C$
10. A
11. D

EXERCISE 7.8

1. $\frac{1}{2}(b^2-a^2)$
2. $\frac{35}{2}$
3. $\frac{19}{3}$
4. $\frac{27}{2}$
5. $e - \frac{1}{e}$
6. $\frac{15+e^8}{2}$

EXERCISE 7.9

1. 2
2. $\log\frac{3}{2}$
3. $\frac{64}{3}$
4. $\frac{1}{2}$
5. 0
6. $e^4(e-1)$

7. $\frac{1}{2} \log 2$

8. $\log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$

9. $\frac{\pi}{2}$

10. $\frac{\pi}{4}$

11. $\frac{1}{2} \log \frac{3}{2}$

12. $\frac{\pi}{4}$

13. $\frac{1}{2} \log 2$

14. $\frac{1}{5} \log 6 + \frac{3}{\sqrt{5}} \tan^{-1} \sqrt{5}$

15. $\frac{1}{2}(e - 1)$

16. $5 - \frac{5}{2} \left(9 \log \frac{5}{4} - \log \frac{3}{2} \right)$

17. $\frac{\pi^4}{1024} + \frac{\pi}{2} + 2$

18. 0

19. $3 \log 2 + \frac{3\pi}{8}$

20. $1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$

21. D

22. C

EXERCISE 7.10

1. $\frac{1}{2} \log 2$

2. $\frac{64}{231}$

3. $\frac{\pi}{2} - \log 2$

4. $\frac{16\sqrt{2}}{15}(\sqrt{2}+1)$

5. $\frac{\pi}{4}$

6. $\frac{1}{\sqrt{17}} \log \frac{21+5\sqrt{17}}{4}$

7. $\frac{\pi}{8}$

8. $\frac{e^2(e^2-2)}{4}$

9. A

10. B

EXERCISE 7.11

1. $\frac{\pi}{4}$

2. $\frac{\pi}{4}$

3. $\frac{\pi}{4}$

4. $\frac{\pi}{4}$

5. 29

6. 9

7. $\frac{1}{(n+1)(n+2)}$

8. $\frac{\pi}{8} \log 2$

9. $\frac{16\sqrt{2}}{15}$

10. $\frac{\pi}{2} \log \frac{1}{2}$

11. $\frac{\pi}{2}$

- 12.** π **13.** 0 **14.** 0 **15.** 0
16. $-\pi \log 2$ **17.** $\frac{a}{2}$ **18.** 5 **20.** C
21. C

MISCELLANEOUS EXERCISE ON CHAPTER 7

- 1.** $\frac{1}{2} \log \left| \frac{x^2}{1-x^2} \right| + C$
- 2.** $\frac{2}{3(a-b)} \left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}} \right] + C$
- 3.** $-\frac{2}{a} \sqrt{\frac{(a-x)}{x}} + C$
- 4.** $-\left(1 + \frac{1}{x^4} \right)^{\frac{1}{4}} + C$
- 5.** $2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log(1+x^{\frac{1}{6}}) + C$
- 6.** $-\frac{1}{2} \log|x+1| + \frac{1}{4} \log(x^2+9) + \frac{3}{2} \tan^{-1} \frac{x}{3} + C$
- 7.** $\sin a \log|\sin(x-a)| + x \cos a + C$
- 8.** $\frac{x^3}{3} + C$
- 9.** $\sin^{-1} \left(\frac{\sin x}{2} \right) + C$
- 10.** $-\frac{1}{2} \sin 2x + C$
- 11.** $\frac{1}{\sin(a-b)} \log \left| \frac{\cos(x+b)}{\cos(x+a)} \right| + C$
- 12.** $\frac{1}{4} \sin^{-1}(x^4) + C$
- 13.** $\log \left(\frac{1+e^x}{2+e^x} \right) + C$
- 14.** $\frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + C$
- 15.** $-\frac{1}{4} \cos^4 x + C$
- 16.** $\frac{1}{4} \log(x^4+1) + C$
- 17.** $\frac{[f(ax+b)]^{n+1}}{a(n+1)} + C$
- 18.** $\frac{-2}{\sin \alpha} \sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$
- 19.** $\frac{2(2x-1)}{\pi} \sin^{-1} \sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + C$

20. $-2\sqrt{1-x} + \cos^{-1}\sqrt{x} + \sqrt{x-x^2} + C$

21. $e^x \tan x + C$

22. $-2\log|x+1| - \frac{1}{x+1} + 3\log|x+2| + C$

23. $\frac{1}{2} \left[x \cos^{-1} x - \sqrt{1-x^2} \right] + C$

24. $-\frac{1}{3} \left(1 + \frac{1}{x^2} \right)^{\frac{3}{2}} \left[\log \left(1 + \frac{1}{x^2} \right) - \frac{2}{3} \right] + C$

25. $\frac{\pi}{e^2}$

26. $\frac{\pi}{8}$

27. $\frac{\pi}{6}$

28. $2\sin^{-1}\frac{(\sqrt{3}-1)}{2}$

29. $\frac{4\sqrt{2}}{3}$

30. $\frac{1}{40} \log 9$

31. $\frac{\pi}{2} - 1$

32. $\frac{\pi}{2}(\pi - 2)$

33. $\frac{19}{2}$

34. $\frac{1}{3} \left(e^2 - \frac{1}{e} \right)$

41. A

42. B

43. D

44. B

EXERCISE 8.1

1. $\frac{14}{3}$

2. $16 - 4\sqrt{2}$

3. $\frac{32 - 8\sqrt{2}}{3}$

4. 12π

5. 6π

6. $\frac{\pi}{3}$

7. $\frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right)$

8. $(4)^{\frac{2}{3}}$

9. $\frac{1}{3}$

10. $\frac{9}{8}$

11. $8\sqrt{3}$

12. A

13. B

EXERCISE 8.2

1. $\frac{\sqrt{2}}{6} + \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$ 2. $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right)$
 3. $\frac{21}{2}$ 4. 4 5. 8
 6. B 7. B

Miscellaneous Exercise on Chapter 8

1. (i) $\frac{7}{3}$ (ii) 624.8
 2. $\frac{1}{6}$ 3. $\frac{7}{3}$ 4. 9 5. 4
 6. $\frac{8a^2}{3m^3}$ 7. 27 8. $\frac{3}{2}(\pi-2)$
 9. $\frac{ab}{4}(\pi-2)$ 10. $\frac{5}{6}$ 11. 2 12. $\frac{1}{3}$
 13. 7 14. $\frac{7}{2}$ 15. $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) + \frac{1}{3\sqrt{2}}$
 16. D 17. C 18. C 19. B

EXERCISE 9.1

1. Order 4; Degree not defined 2. Order 1; Degree 1
 3. Order 2; Degree 1 4. Order 2; Degree not defined
 5. Order 2; Degree 1 6. Order 3; Degree 2
 7. Order 3; Degree 1 8. Order 1; Degree 1
 9. Order 2; Degree 1 10. Order 2; Degree 1
 11. D 12. A

EXERCISE 9.2

11. D 12. D

EXERCISE 9.3

1. $y'' = 0$
 2. $xy'' + x(y')^2 - y y' = 0$
 3. $y'' - y' - 6y = 0$
 4. $y'' - 4y' + 4y = 0$
 5. $y'' - 2y' + 2y = 0$
 6. $2xyy' + x^2 = y^2$
 7. $xy' - 2y = 0$
 8. $xyy'' + x(y')^2 - yy' = 0$
 9. $xyy'' + x(y')^2 - yy' = 0$
 10. $(x^2 - 9)(y')^2 + x^2 = 0$
 11. B
 12. C

EXERCISE 9.4

1. $y = 2 \tan \frac{x}{2} - x + C$
 2. $y = 2 \sin(x + C)$
 3. $y = 1 + Ae^{-x}$
 4. $\tan x \tan y = C$
 5. $y = \log(e^x + e^{-x}) + C$
 6. $\tan^{-1} y = x + \frac{x^3}{3} + C$
 7. $y = e^{cx}$
 8. $x^{-4} + y^{-4} = C$
 9. $y = x \sin^{-1} x + \sqrt{1-x^2} + C$
 10. $\tan y = C(1 - e^x)$
 11. $y = \frac{1}{4} \log[(x+1)^2(x^2+1)^3] - \frac{1}{2} \tan^{-1} x + 1$
 12. $y = \frac{1}{2} \log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2} \log \frac{3}{4}$
 13. $\cos\left(\frac{y-2}{x}\right) = a$
 14. $y = \sec x$
 15. $2y - 1 = e^x(\sin x - \cos x)$
 16. $y - x + 2 = \log(x^2(y+2)^2)$
 17. $y^2 - x^2 = 4$
 18. $(x+4)^2 = y+3$
 19. $(63t+27)^{\frac{1}{3}}$
 20. 6.93%
 21. Rs 1648
 22. $\frac{2 \log 2}{\log\left(\frac{11}{10}\right)}$
 23. A

EXERCISE 9.5

1. $(x-y)^2 = Cx e^{\frac{-y}{x}}$
 2. $y = x \log|x| + Cx$

3. $\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2} \log(x^2 + y^2) + C$ 4. $x^2 + y^2 = Cx$

5. $\frac{1}{2\sqrt{2}} \log \left| \frac{x+\sqrt{2}y}{x-\sqrt{2}y} \right| = \log|x| + C$ 6. $y + \sqrt{x^2 + y^2} = Cx^2$

7. $xy \cos \left| \frac{y}{x} \right| = C$

8. $x \left[1 - \cos \left(\frac{y}{x} \right) \right] = C \sin \left(\frac{y}{x} \right)$

9. $cy = \log \left| \frac{y}{x} \right| - 1$

10. $ye^{\frac{x}{y}} + x = C$

11. $\log(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} = \frac{\pi}{2} + \log 2$

12. $y + 2x = 3x^2 y$

13. $\cot \left(\frac{y}{x} \right) = \log|ex|$

14. $\cos \left(\frac{y}{x} \right) = \log|ex|$

15. $y = \frac{2x}{1 - \log|x|} (x \neq 0, x \neq e)$

16. C

EXERCISE 9.6

1. $y = \frac{1}{5}(2\sin x - \cos x) + C e^{-2x}$ 2. $y = e^{-2x} + C e^{-3x}$

3. $xy = \frac{x^4}{4} + C$

4. $y(\sec x + \tan x) = \sec x + \tan x - x + C$

5. $y = (\tan x - 1) + C e^{-\tan x}$

6. $y = \frac{x^2}{16}(4 \log|x| - 1) + C x^{-2}$

7. $y \log x = \frac{-2}{x}(1 + \log|x|) + C$

8. $y = (1+x)^{-1} \log|\sin x| + C(1+x^2)^{-1}$

9. $y = \frac{1}{x} - \cot x + \frac{C}{x \sin x}$

10. $(x + y + 1) = C e^y$

11. $x = \frac{y^2}{3} + \frac{C}{y}$

12. $x = 3y^2 + Cy$

13. $y = \cos x - 2 \cos^2 x$

14. $y(1+x^2) = \tan^{-1} x - \frac{\pi}{4}$

15. $y = 4 \sin^3 x - 2 \sin^2 x$

16. $x + y + 1 = e^x$

17. $y = 4 - x - 2 e^x$

18. C

19. D

Miscellaneous Exercise on Chapter 9

1. (i) Order 2; Degree 1 (ii) Order 1; Degree 3
 (iii) Order 4; Degree not defined

3. $y' = \frac{2y^2 - x^2}{4xy}$

5. $(x + yy')^2 = (x - y)^2 (1 + (y')^2)$

6. $\sin^{-1} y + \sin^{-1} x = C$

8. $\cos y = \frac{\sec x}{\sqrt{2}}$

9. $\tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$

10. $e^y = y + C$

11. $\log |x - y| = x + y + 1$

12. $ye^{2\sqrt{x}} = (2\sqrt{x} + C)$

13. $y \sin x = 2x^2 - \frac{\pi^2}{2} (\sin x \neq 0)$

14. $y = \log \left| \frac{2x+1}{x+1} \right|, x \neq -1$

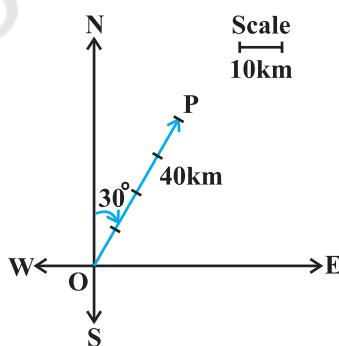
15. 31250

16. C

17. C

EXERCISE 10.1

1. In the adjoining figure, the vector \overrightarrow{OP} represents the required displacement.



- 2.** (i) scalar (ii) vector (iii) scalar (iv) scalar (v) scalar
 (vi) vector
- 3.** (i) scalar (ii) scalar (iii) vector (iv) vector (v) scalar
- 4.** (i) Vectors \vec{a} and \vec{b} are coinitial
 (ii) Vectors \vec{b} and \vec{d} are equal
 (iii) Vectors \vec{a} and \vec{c} are collinear but not equal
- 5.** (i) True (ii) False (iii) False (iv) False

EXERCISE 10.2

- 1.** $|\vec{a}| = \sqrt{3}, |\vec{b}| = \sqrt{62}, |\vec{c}| = 1$
- 2.** An infinite number of possible answers.
- 3.** An infinite number of possible answers.
- 4.** $x = 2, y = 3$
- 5.** -7 and 6 ; $-7\hat{i}$ and $6\hat{j}$
- 6.** $-4\hat{j} - \hat{k}$
- 7.** $\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$
- 8.** $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$
- 9.** $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$
- 10.** $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$
- 12.** $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$
- 13.** $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$
- 15.** (i) $-\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$ (ii) $-3\hat{i} + 3\hat{k}$
- 16.** $3\hat{i} + 2\hat{j} + \hat{k}$
- 18.** (C)
- 19.** (B), (C), (D)

EXERCISE 10.3

- 1.** $\frac{\pi}{4}$
- 2.** $\cos^{-1}\left(\frac{5}{7}\right)$
- 3.** 0
- 4.** $\frac{60}{\sqrt{114}}$
- 6.** $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$
- 7.** $6|\vec{a}|^2 + 11\vec{a} \cdot \vec{b} - 35|\vec{b}|^2$
- 8.** $|\vec{a}| = 1, |\vec{b}| = 1$
- 9.** $\sqrt{13}$
- 10.** 8

12. Vector \vec{b} can be any vector 13. $\frac{-3}{2}$
14. Take any two non-zero perpendicular vectors \vec{a} and \vec{b}
15. $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$ 18. (D)

EXERCISE 10.4

1. $19\sqrt{2}$ 2. $\pm\frac{2}{3}\hat{i} \mp\frac{2}{3}\hat{j} \mp\frac{1}{3}\hat{k}$ 3. $\frac{\pi}{3}; \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$
5. $3, \frac{27}{2}$ 6. Either $|\vec{a}|=0$ or $|\vec{b}|=0$
8. No; take any two nonzero collinear vectors
9. $\frac{\sqrt{61}}{2}$ 10. $15\sqrt{2}$ 11. (B) 12. (C)

Miscellaneous Exercise on Chapter 10

1. $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$
2. $x_2 - x_1, y_2 - y_1, z_2 - z_1; \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
3. $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$
4. No; take \vec{a} , \vec{b} and \vec{c} to represent the sides of a triangle.
5. $\pm\frac{1}{\sqrt{3}}$ 6. $\frac{3}{2}\sqrt{10}\hat{i} + \frac{\sqrt{10}}{2}\hat{j}$ 7. $\frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$
8. 2 : 3 9. $3\vec{a} + 5\vec{b}$ 10. $\frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k}); 11\sqrt{5}$
12. $\frac{1}{3}(160\hat{i} - 5\hat{j} + 70\hat{k})$ 13. $\lambda = 1$ 16. (B)
17. (D) 18. (C) 19. (B)

EXERCISE 11.1

- 1.** $0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ **2.** $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$ **3.** $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$
5. $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{17}; \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}; \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

EXERCISE 11.2

4. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$, where λ is a real number

5. $\vec{r} = 2\hat{i} - \hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and cartesian form is

$$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{-1}$$

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

7. $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

8. Vector equation of the line: $\vec{r} = \lambda(5\hat{i} - 2\hat{j} + 3\hat{k})$;

$$\text{Cartesian equation of the line: } \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

9. Vector equation of the line: $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda(11\hat{k})$

$$\text{Cartesian equation of the line: } \frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}$$

10. (i) $\theta = \cos^{-1}\left(\frac{19}{21}\right)$ (ii) $\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

11. (i) $\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$ (ii) $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

12. $p = \frac{70}{11}$ **14.** $\frac{3\sqrt{2}}{2}$ **15.** $2\sqrt{29}$

16. $\frac{3}{\sqrt{19}}$ **17.** $\frac{8}{\sqrt{29}}$

EXERCISE 11.3

- 1.** (a) $0, 0, 1; 2$ (b) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}$
- (c) $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}; \frac{5}{\sqrt{14}}$ (d) $0, 1, 0; \frac{8}{5}$
- 2.** $\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7$
- 3.** (a) $x + y - z = 2$ (b) $2x + 3y - 4z = 1$
 (c) $(s - 2t)x + (3 - t)y + (2s + t)z = 15$
- 4.** (a) $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29} \right)$ (b) $\left(0, \frac{18}{25}, \frac{24}{25} \right)$
 (c) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$ (d) $\left(0, \frac{-8}{5}, 0 \right)$
- 5.** (a) $[\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0; x + y - z = 3$
 (b) $[\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0; x - 2y + z + 1 = 0$
- 6.** (a) The points are collinear. There will be infinite number of planes passing through the given points.
 (b) $2x + 3y - 3z = 5$
- 7.** $\frac{5}{2}, 5, -5$ **8.** $y = 3$ **9.** $7x - 5y + 4z - 8 = 0$
- 10.** $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$ **11.** $x - z + 2 = 0$
- 12.** $\cos^{-1} \frac{15}{\sqrt{731}}$
- 13.** (a) $\cos^{-1} \left(\frac{2}{5} \right)$ (b) The planes are perpendicular
 (c) The planes are parallel (d) The planes are parallel
 (e) 45°
- 14.** (a) $\frac{3}{13}$ (b) $\frac{13}{3}$
 (c) 3 (d) 2

Miscellaneous Exercise on Chapter 11

3. 90°

4. $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

5. 0°

6. $k = \frac{-10}{7}$

7. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$

8. $x + y + z = a + b + c$

9. 9

10. $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

11. $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

12. $(1, -2, 7)$

13. $7x - 8y + 3z + 25 = 0$

14. $p = \frac{3}{2}$ or $\frac{11}{6}$ or $\frac{7}{3}$

15. $y - 3z + 6 = 0$

16. $x + 2y - 3z - 14 = 0$

17. $33x + 45y + 50z - 41 = 0$

18. 13

19. $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$

22. D

20. $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

23. B

EXERCISE 12.11. Maximum $Z = 16$ at $(0, 4)$ 2. Minimum $Z = -12$ at $(4, 0)$ 3. Maximum $Z = \frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$ 4. Minimum $Z = 7$ at $\left(\frac{3}{2}, \frac{1}{2}\right)$ 5. Maximum $Z = 18$ at $(4, 3)$ 6. Minimum $Z = 6$ at all the points on the line segment joining the points $(6, 0)$ and $(0, 3)$.7. Minimum $Z = 300$ at $(60, 0)$;Maximum $Z = 600$ at all the points on the line segment joining the points $(120, 0)$ and $(60, 30)$.

- 8.** Minimum $Z = 100$ at all the points on the line segment joining the points $(0, 50)$ and $(20, 40)$;
 Maximum $Z = 400$ at $(0, 200)$
- 9.** Z has no maximum value
- 10.** No feasible region, hence no maximum value of Z .

EXERCISE 12.2

- 1.** Minimum cost = Rs 160 at all points lying on segment joining $\frac{8}{3}, 0$ and $2, \frac{1}{2}$.
- 2.** Maximum number of cakes = 30 of kind one and 10 cakes of another kind.
- 3.** (i) 4 tennis rackets and 12 cricket bats
 (ii) Maximum profit = Rs 200
- 4.** 3 packages of nuts and 3 packages of bolts; Maximum profit = Rs 73.50.
- 5.** 30 packages of screws A and 20 packages of screws B; Maximum profit = Rs 410
- 6.** 4 Pedestal lamps and 4 wooden shades; Maximum profit = Rs 32
- 7.** 8 Souvenir of types A and 20 of Souvenir of type B; Maximum profit = Rs 160.
- 8.** 200 units of desktop model and 50 units of portable model; Maximum profit = Rs 1150000.
- 9.** Minimise $Z = 4x + 6y$
 subject to $3x + 6y \geq 80$, $4x + 3y \geq 100$, $x \geq 0$ and $y \geq 0$, where x and y denote the number of units of food F_1 and food F_2 respectively; Minimum cost = Rs 104
- 10.** 100 kg of fertiliser F_1 and 80 kg of fertiliser F_2 ; Minimum cost = Rs 1000
- 11.** (D)

Miscellaneous Exercise on Chapter 12

- 1.** 40 packets of food P and 15 packets of food Q; Maximum amount of vitamin A = 285 units.
- 2.** 3 bags of brand P and 6 bags of brand Q; Minimum cost of the mixture = Rs 1950
- 3.** Least cost of the mixture is Rs 112 (2 kg of Food X and 4 kg of food Y).

- 40 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 136000.
 - From A : 10,50, 40 units; From B: 50,0,0 units to D, E and F respectively and minimum cost = Rs 510
 - From A: 500, 3000 and 3500 litres; From B: 4000, 0, 0 litres to D, E and F respectively; Minimum cost = Rs 4400
 - 40 bags of brand P and 100 bags of brand Q; Minimum amount of nitrogen = 470 kg.
 - 140 bags of brand P and 50 bags of brand Q; Maximum amount of nitrogen = 595 kg.
 - 800 dolls of type A and 400 dolls of type B; Maximum profit = Rs 16000

EXERCISE 13.1

$$1. \quad P(E|F) = \frac{2}{3}, \quad P(F|E) = \frac{1}{3}$$

$$2. P(A|B) = \frac{16}{25}$$

3. (i) 0.32

(ii) 0.64

(iii) 0.98

$$4. \quad \frac{11}{26}$$

5. (i) $\frac{4}{11}$

(ii) $\frac{4}{5}$

(iii) $\frac{2}{3}$

6. (i) $\frac{1}{2}$

(ii) $\frac{3}{7}$

(iii) $\frac{6}{\pi}$

7. (j) 1

(ii) 0

8. $\frac{1}{6}$

9. 1

10. (a) $\frac{1}{3}$, (b) $\frac{1}{9}$

11. (i) $\frac{1}{2}, \frac{1}{3}$

$$(ii) \quad \frac{1}{2}, \frac{2}{3}$$

(iii) $\frac{3}{4}, \frac{1}{4}$

12. (i) $\frac{1}{2}$

(ii) $\frac{1}{2}$

13.

14. $\frac{1}{15}$

15. 0

16. C

EXERCISE 13.2

1. $\frac{3}{25}$

2. $\frac{25}{102}$

3. $\frac{44}{91}$

4. A and B are independent

5. A and B are not independent

6. E and F are not independent

7. (i) $p = \frac{1}{10}$

(ii) $p = \frac{1}{5}$

8. (i) 0.12

(ii) 0.58

(iii) 0.3

(iv) 0.4

9. $\frac{3}{8}$

10. A and B are not independent

11. (i) 0.18 (ii) 0.12 (iii) 0.72 (iv) 0.28

12. $\frac{7}{8}$

13. (i) $\frac{16}{81}$, (ii) $\frac{20}{81}$, (iii) $\frac{40}{81}$

14. (i) $\frac{2}{3}$, (ii) $\frac{1}{2}$

15. (i), (ii)

16. (a) $\frac{1}{5}$, (b) $\frac{1}{3}$, (c) $\frac{1}{2}$

17. D

18. B

EXERCISE 13.3

1. $\frac{1}{2}$

2. $\frac{2}{3}$

3. $\frac{9}{13}$

4. $\frac{12}{13}$

5. $\frac{22}{133}$

6. $\frac{4}{9}$

7. $\frac{1}{52}$

8. $\frac{1}{4}$

9. $\frac{2}{9}$

10. $\frac{8}{11}$

11. $\frac{5}{34}$

12. $\frac{11}{50}$

13. A

14. C

EXERCISE 13.4

1. (ii), (iii) and (iv)

2. $X = 0, 1, 2$; yes3. $X = 6, 4, 2, 0$

4. (i)	X	0	1	2
	P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

(ii)	X	0	1	2	3
	P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(iii)	X	0	1	2	3	4
	P(X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

5. (i)

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii)

X	0	1
P(X)	$\frac{25}{36}$	$\frac{11}{36}$

6.

X	0	1	2	3	4
P(X)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

7.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{6}{16}$	$\frac{1}{16}$

8. (i) $k = \frac{1}{10}$ (ii) $P(X < 3) = \frac{3}{10}$ (iii) $P(X > 6) = \frac{17}{100}$

(iv) $P(0 < X < 3) = \frac{3}{10}$

9. (a) $k = \frac{1}{6}$ (b) $P(X < 2) = \frac{1}{2}$, $P(X \leq 2) = 1$, $P(X \geq 2) = \frac{1}{2}$

10. 1.5 11. $\frac{1}{3}$ 12. $\frac{14}{3}$

13. $\text{Var}(X) = 5.833$, $S.D = 2.415$

14.

X	14	15	16	17	18	19	20	21
P(X)	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Mean = 17.53, $\text{Var}(X) = 4.78$ and $S.D(X) = 2.19$

15. $E(X) = 0.7$ and $\text{Var}(X) = 0.21$ 16. B 17. D

EXERCISE 13.5

1. (i) $\frac{3}{32}$ (ii) $\frac{7}{64}$ (iii) $\frac{63}{64}$

$$2. \quad \frac{25}{216} \qquad 3. \quad \left(\frac{29}{20}\right)\left(\frac{19}{20}\right)^9$$

4. (i) $\frac{1}{1024}$ (ii) $\frac{45}{512}$ (iii) $\frac{243}{1024}$

6. $\left(\frac{9}{10}\right)^4$ 7. $\left(\frac{1}{2}\right)^{20} [20C_{12} + {}^{20}C_{13} + \dots + {}^{20}C_{20}]$

$$9. \quad \frac{11}{243}$$

10. (a) $1 - \left(\frac{99}{100}\right)^{50}$ (b) $\frac{1}{2} \left(\frac{99}{100}\right)^{49}$ (c) $1 - \frac{149}{100} \left(\frac{99}{100}\right)^{49}$

11. $\frac{7}{12} \left(\frac{5}{6}\right)^5$ 12. $\frac{35}{18} \left(\frac{5}{6}\right)^4$ 13. $\frac{22 \times 9^3}{10^{11}}$

14. C

Miscellaneous Exercise on Chapter 13

1. (i) 1 (ii) 0

2. (i) $\frac{1}{3}$  (ii) $\frac{1}{2}$

$$3. \quad \frac{20}{21}$$

$$4. \quad 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

5. (i) $\left(\frac{2}{5}\right)^6$ (ii) $7\left(\frac{2}{5}\right)^4$ (iii) $1 - \left(\frac{2}{5}\right)^6$ (iv) $\frac{864}{3125}$

6. $\frac{5^{10}}{2 \times 6^9}$

7. $\frac{625}{23328}$

8. $\frac{2}{7}$

9. $\frac{31}{9} \left(\frac{2}{3} \right)^4$

10. $n \geq 4$

11. $\frac{-91}{54}$

12. $\frac{1}{15}, \frac{2}{5}, \frac{8}{15}$

13. $\frac{14}{29}$

14. $\frac{3}{16}$

15. (i) 0.5 (ii) 0.05

16. $\frac{16}{31}$

17. A**18.** C**19.** B